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THE JOURNAL OF PHILOSOPHY

A BUDGET OF CROSS-TYPE INFERENCES, OR INVENTION IS THE MOTHER OF NECESSITY[1]

FROM what is contingent nothing necessary follows. And from what is necessary nothing contingent follows. Let us call this "the Hume-Leibniz dictum." These theses never occur full-blown in Hume or Leibniz, but the dictum is implicit within the philosophies history has come to associate with these thinkers. Thus Hume writes:

> Relations of ideas [are] either intuitively or demonstratively certain . . .[however,] matters of fact . . . are not ascertained in the same manner; . . . The contrary of every matter of fact is still possible; because *it can never imply a contradiction* . . . (*Enquiry*, IV, I; my italics.)

And Leibniz says:

> There are also two kinds of truths, those of reasoning and those of fact. Truths of reasoning are necessary and their opposite is impossible; truths of fact are contingent and their opposite is possible. (*Monadology*, 33.)

Stressing these differences in type between necessary and contingent propositions is proceeding toward a rejection, by Hume and by Leibniz, of the possibility of any genuine, nondegenerate, cross-type inference. For, if P states a possible matter of fact, so does $\sim$P. But then neither P nor $\sim$P implies what is logically false. This much cross-typicality is clearly barred by Hume.

What about inferences from contingencies to what is logically true, however? On this, Leibniz says:

> According to the usage of logicians *the conclusion follows the weakest of the premises* [*conclusio sequitur partem debiliorem*] and cannot have more certainty than they. (*New Essays*, IV, ix, 14.)

Hence, if Q follows from a set of premises of which one (P) is contingent, then Q cannot have more certainty than P. Therefore, anything entailed by P must also be contingent—so suggests Leibniz.

[1] I am indebted to Richard Smyth, Roger Buck, and Grover Maxwell for stimulation and criticism during the writing of this paper.

So Hume and Leibniz appear to show sympathy for the contention that from what is contingent nothing necessary follows.

Concerning the second half of the dictum, Hume and Leibniz are again far from explicit. Nonetheless, let Q be a "truth of fact"; then its "opposite is possible"; that is, $\Diamond \sim Q$. Let P be a "truth of reason"; then its "opposite is impossible"; that is, $\sim \Diamond \sim P$. Suppose now that this P entails this Q; that is, $(\sim \Diamond \sim P) \dashv (\Diamond \sim Q)$. This runs counter to a theorem in Lewis and Langford (*Symbolic Logic*, 18.53), to wit:

$$[(P \dashv Q) \cdot (\sim \Diamond \sim P)] \dashv (\sim \Diamond \sim Q)$$

Hume and Leibniz probably would have accepted this principle. Indeed, Hume's claim that "the contrary of [a] matter of fact . . . can never imply a contradiction" transforms (by contraposition) into the claim that "from what is necessary nothing contingent follows." But strict alignment with our texts invites us to call our dictum "the Hume-Leibniz-Lewis-Langford dictum" (HLLL), which, to express it in yet different terms, will read:

> Any claim whose negation is consistent can have followed only from premises at least one of which has a consistent negation. And any claim entailed by premises whose negations are inconsistent must itself have an inconsistent negation.

In what philosophically significant sense a claim with an inconsistent negation can have followed from premises with consistent negations, this it will be our task to discover. Incidentally, Lewis and Langford, although denying that necessary propositions entail contingent propositions, consistently affirm that the former are *entailed by* the latter. This must be dealt with in the sequel.

Although never defended in full generality, this dictum has been valuable to modern philosophy. Qualitatively put, it states that no mere inspection of facts will secure certainty for a contingent conclusion. No amount of formal rigor will secure the factual truth of a conclusion unless contingent (and hence vulnerable) statements are fed into the premises. This logical blade has axed forests of wooden metaphysics to the ground. For, when it is seen that experience is insufficient to establish capital-lettered, necessary claims about the Cosmos—and when armchair reasoning is seen to be insufficient for determining what will be found in the world—much of traditional metaphysics collapses. Thus, the HLLL dictum cleared a new view of the nature of philosophical inquiry.

Notwithstanding its methodological utility, however, the dictum's exposition must be sharpened. This is because we regularly encounter valid entailments that appear to move across types. If, in fact, they *do* so move, then Hume and Leibniz and Lewis and Langford must be put to the question. Moreover, no general condemnation of speculative metaphysics as consisting of cross-type inferences will have any validity unless the apparent cross-typicali-

ties of logic and philosophy are reappraised. Of course, our venture here may fail entirely. After the following analyses it may still be necessary to concede that nondegenerate cross-type inferences do exist; if speculative metaphysics is suspect, it may have to be for reasons different from those set out here. But then the failure may be remarked in detail by others, and a rarely discussed perplexity will at least have been canvassed.

It is my objective to set out a "budget" of these putative cross-type inferences. The next section (II) will delineate how from contingent propositions necessary propositions can apparently be inferred. After that it will be indicated how from necessary propositions contingent propositions can apparently be inferred (III). Then an attempt will be made to resolve the perplexities proposed within each of these inferences (IV). Finally, a restatement of the HLLL dictum will be sought, such that it does not encroach upon certain valid inference-schemata (V).

Before plunging into our budget, one or two cautions are in order. One might resist the HLLL dictum not because it is open to counter-instances, but because it is ineffective. P is necessary if its negation leads to contradictions; that this is so depends on having a technique for detecting inconsistencies within a potential infinitude of other consequences. But it is never $\sim$P *simpliciter* from which we generate inconsistency; rather, it is $\sim$P in conjunction with other unstated assumptions (e.g., rules of deduction; axioms—e.g., of reducibility, of infinity; principles—e.g., of mathematical induction, etc.). *If* contradiction follows from operations on $\sim$P, then either (1) $\sim$P is self-contradictory, *or* (2) the unstated assumptions contain contradictions, *or* (3) the "operator" is a poor logician. In principle, these other alternatives, (2) and (3), are never eliminable. But in practice they are. "Jones is a married bachelor" is a paradigm of what counts as a contradiction within a language—however true it is that this verdict involves contingent assumptions about the utility and permanence of certain explicit definitions and linguistic conventions within that language.

Our concern here is not with *establishing that* a given, particular claim is inconsistent. Difficulties such as those elaborated by Brouwer and Quine have bulked large within a literature quite different from what follows. Our present concern is with *what follows from* assuming a given claim to be inconsistent. Doubtless this assumption makes sense. Even if not one actual claim within a language L were ever demonstrated inconsistent in L, it would still make sense to elaborate the characteristics of a proposition *assumed* to be incontestably inconsistent within L. We can never demonstrate the existence of an ideal gas, but we know its properties intimately. So the esoteric difficulties which trouble finitistic logicians and which appear within recent discussions of analyticity will not deter us.

A final caution: Our guiding principle is to the effect that cross-type inferences are somehow semantically untenable. For, assume P to be an incontestably necessary claim (in that, within L, $\sim$P *has already been discovered* to generate something of the form $F \cdot \sim F$). And assume that Q is incontestably contingent (in that, within L, $\sim$Q generates nothing of the form $F \cdot \sim F$—nor does Q either, of course); indeed, suppose that $\sim Q \dashv (F \cdot \sim F)$ and $Q \dashv (F \cdot \sim F)$ themselves have been discovered to be inconsistent: nothing could better establish the consistency of Q, and of $\sim$Q, and hence the contingency of both. Then assume that P entails Q. But then,

$$(P \dashv Q) \equiv (\sim Q \dashv \sim P)$$

This is semantically absurd, since it states that an inference from what is necessary to what is contingent *is logically equivalent to* an inference from what is contingent to what is impossible—*reductio ad absurdum*. I should prefer to describe this as showing that, should an L-determinate P be taken to entail a contingent Q, this would have to be logically equivalent to a contingent $\sim$Q entailing an L-determinate $\sim$P—*reductio ad absurdum*. Two inferences could not possibly be more different. *Inferring from* what is contingent must be profoundly different from *inferring from* what is L-determinate. So, cross-type inferences in general are untenable, and the specific assumption that one might infer from a necessary to a contingent claim ends up in a contradiction. How, then, can we account for the cadidates for inferential cross-typicality that follow?

II. Inferences from Contingency to Necessity

1. "If it is contingently true that P and contingently true that Q, then the claim that P is contingently true and the claim that Q is contingently true are consistent claims." (Cf. Lewis and Langford, *Symbolic Logic*, 17.1.)[2]

[2] Comparisons with Lewis and Langford serve only as references, not as justifications, for our examples. 17.1 reads: "$P \cdot Q \dashv P \circ Q$." The example above is cast (at least) one metalanguage above this: $(P_{ct} \cdot Q_{ct}) \dashv (P_{ct} \circ Q_{ct})$. *This* reads: "That P is contingently true and that Q is contingently true jointly entail that the contingent truth of P is compatible with Q's being contingently true." References to Lewis and Langford are thus of analogical interest merely. A different notation in our example could have brought 17.1 directly into the argument: for instance, let

R = P is contingently true
S = Q is contingently true

Then example 1 would read: "If R and S obtain, then R and S are compatible"—which is a valid substitution instance of Lewis and Langford 17.1. The subscript notation set out in this footnote will be developed, however; it has advantages that outweigh its unfamiliarity.

The claim that P·Q is contingently true, this is itself contingent. P·Q need not have been true. But that the contingent truth of P is *consistent* with the contingent truth of Q, if this is true at all, it could not but have been true. Or else the claim is logically false. Settling whether the contingent truth of P is consistent with the contingent truth of Q, itself requires no contingent inquiry, only reflection. Whether the contingent truth of P·Q entails anything of the form F·∼F is a formal question. Yet we *do* justifiably infer the latter (i.e., the mutual consistency) from the former (i.e., the joint contingent truth). From observing two phenomena, we do infer that their descriptions are consistent. And from noting that the descriptions of those phenomena are contingently true, a higher-level consistency would also seem to follow. This higher-level consistency is what concerns us here, since, *prima facie*, our example constitutes an inference from what is contingent to what is necessary.[3]

2. "That P is contingently true entails the claim that contingent P is logically possible." (Cf. Lewis and Langford, 18.4.)

That P is logically possible just *is* the claim that P is self-consistent. This, again, is a formal claim—the claim that P entails nothing of the form F·∼F. This is settled by reflection alone. However, that P contingently obtains is itself a contingent claim. It makes sense to assert that ∼P obtains. Which of the two ("P contingently obtains," or "∼P contingently obtains") states fact, requires contingent inquiry to settle. Still, inferring from "P is contingently true" to "P is logically possible" seems valid. Hence, another putative counter-instance to the HLLL dictum.

3. "That ∼P is contingently true entails the claim that ∼P is logically possible." (Cf. Lewis and Langford, 18.44.)

The analysis appropriate to example 2 (above) applies *mutatis mutandis* in this case.

4. "If it is true both that P is contingently true *and* that Q is necessarily true, we can, from this conjunction, infer that Q is necessarily true." (Cf. Lewis and Langford, 11.2.)

[3] This example may still contain an ambiguity that will affect future analyses. Two different things *might* have been intended by our example above:

(a) If P·Q is contingently true, then $P_{et} \circ Q_{et}$,

or

(b) Where P, Q are contingent, P·Q implies P ○ Q.

In (a), we infer that P's contingent truth is consistent with the contingent truth of Q *from* "P·Q is contingently true."

In (b), we infer that P is consistent with Q *from* "P·Q"—it being understood in this second case that both P and Q are contingent. Interpretation (a) is the one we use throughout.

Again, that P is contingently true is itself a contingent claim. But that Q is necessarily true—this assertion is either necessarily true or necessarily false. It is certainly L-determinate. Its credentials are settled solely by reflection. Our example conjoins this contingent and this necessary claim; from this conjunction it follows that Q is necessarily true. Hence, a necessary conclusion from a contingent conjunction.

Nothing in classical or modern logic forbids conjoining necessary and contingent claims. Indeed, textbook discussions of the Square of Opposition usually include sample statements such as "All bachelors are male, *and* there are in fact bachelors"; that is, $(x)(B_x \supset M_x) \cdot (\exists x)(B_x)$. This can only be contingently true. It cannot be established by reflection alone; some appeal to the facts that support $(\exists x)(B_x)$ is required to establish the truth of the whole conjunction. Still, by detachment, from a conjunction one can infer to any constituent conjunct. Hence, from "All bachelors are male *and* there are bachelors" (contingently true) one can infer to "All bachelors are male" (necessarily true). The conclusion is incontestably necessary. Yet that from which it is inferred is contingent. Moreover, the inference is legitimate.

Note that, despite the ink poured over the Square of Opposition, no philosopher has noted that the *modified* A-form demanded by contemporary logicians (e.g., "All bachelors are male, *and* there are bachelors") sometimes conjoins a necessary with a contingent claim. This sanctions a cross-type inference. The ultimate consequences of this could conceivably prove more uncomfortable than ever did any existentially important inference from an *un*modified A-form (e.g., "All bachelors are male, so there exists a male bachelor").

Consider also the efforts of J. S. Mill to "justify" mathematics *via* experience, an enterprise far from dead even now. Suppose all mathematical propositions that are indisputably necessary to be compressed within the single (internally conjunctive) proposition Q. Suppose all of physical theory (Θ) then to be represented as a "conjunction" of contingent elements P and the necessary elements Q just referred to. Then, from the contingent truth of Θ (i.e., contingent-P-and-necessary-Q,) one could legitimately infer to Q. Attacks on Mill's argument might be weakened somewhat if the inference before us is valid. Mill's case would appear to be more than not obviously unsound; it seems to be sanctionable in a well-established form of inference.

5. A degenerate analogon of this inference is produced thus: "P is contingently true; *so* P is contingently true *and* Q is necessarily true." Thus, "That there are bachelors is contingently true; *so*

that there are bachelors is contingently true *and* that all bachelors are male is necessarily true."

A necessity can be conjoined with any contingent truth, the result being a contingent truth. Since one can infer P from P, one can also infer from P to P *and* (necessary) Q. But, when P implies a conjunction, it also implies either conjunct separately; i.e., if $P \rightarrow R \cdot S$, then $(P \rightarrow R) \cdot (P \rightarrow S)$. And, if $P \rightarrow P \cdot Q$, then $(P \rightarrow P) \cdot (P \rightarrow Q)$. Via this argument, then, contingent P implies necessary Q. Again, this is cross-type.

III. Inferences from Necessity to Contingency

1. Consider the following abbreviations:

F_1: At present B's are regularly followed by C's.
F_2: In the past B's have always been followed by C's.
F_3: All extant theories do, in fact, relate the concepts "being a B" and "being followed by a C."

(Here, this last constitutes no more than a contingent, factual reference to what presently accepted theories *do actually do*. This merely describes a property all theories concerning B's and C's now have. No "inference permit" is being smuggled in.)

F_4: There are no known experimental or theoretical grounds for expecting anything other than a C to follow a B.

Entertain now the following inference:

(a) "That F_1, F_2, F_3, F_4 obtain is good inductive reason for expecting C."

entails

(b) "That F_4 obtains is *a* good inductive reason for expecting C."

The inference from (a) to (b) appears to be cross-type: a necessary proposition entailing a contingent one. When F_1 to F_4 obtain, it would be *irrational* not to infer to C after witnessing another B. F_1 to F_4 obtaining constitutes the *meaning* of "having good inductive reason for expecting C whenever a B occurs." To understand what F_1, F_2, F_3, and F_4 mean and to understand what (a) asserts just *is* to realize that (a) could not but be true. Its semantic content guarantees its truth.

But (b) makes a contingent claim. For example, F_2's obtaining is *a* good inductive reason for expecting C, *contingent upon* F_1, F_3, and F_4 actually obtaining. To expect C because of F_2 alone would be like expecting one's brakes to hold just because they had held during their last 10,000 applications. (From only this we are more justified in expecting the brakes *not* to hold.) Only when further

conditions (F_1, F_3, and F_4) are met will F_2 in this context constitute a good reason for expecting C.

Nonetheless, "That F_1, F_2, F_3, F_4 obtain is good inductive reason for expecting C" surely *entails* "That F_2 obtains is a good inductive reason for expecting C"? Thus our inference apparently runs across types: from what is necessary to what is contingent.

2. Our final example requires some technical apparatus. "If contingent P entails contingent Q, then in fact either Q obtains or P does not." That is, if contingent P *strictly implies* contingent Q, then P *materially implies* Q. (Cf. Lewis and Langford, 14.1.) In symbols:

$$[(P_{ct} \dashv Q_{ct}) \dashv (P_{ct} \supset Q_{ct})]^n$$

As before, the subscript "c" signifies "contingent"; "t" signifies "true"; "n" will signify "necessary". Note again that these denote *actual components within* our examples of inference, not merely metalinguistic comments on the inferences; the corresponding superscripts on the other hand will serve as such comments. "P_{ct}" thus stands for "the proposition P is contingently true"; "Q_{nt}" would stand for "the proposition Q is necessarily true."

The formula above then reads:

That P is contingently true entails that Q is contingently true; therefore, if P is contingently true, Q is also contingently true.

(The final superscript "n" is thus an aside to the effect that the entire inference is necessarily valid.)

Now let P and Q take contingent values; for instance, let P = "Jones is a bachelor," and Q = "Jones is male." Then $P_{ct} \dashv Q_{ct}$ reads: "That *Jones is a bachelor* is contingently true entails that *Jones is male* is contingently true"—and all this is necessarily true.

The material implication, however, would seem to require a different analysis. All that "$P_{ct} \supset Q_{ct}$" means is that *in fact* $\sim P_{ct} \vee Q_{ct}$. (Cf. *Principia Mathematica* *1.01.) Now $\sim P_{ct} \vee Q_{ct}$ is clearly contingent. A disjunction of contingent claims is itself contingent (with the obvious exceptions of $P \vee \sim P$ and its derivatives, $P \vee \sim P \cdot Q$, $P \vee \sim P \cdot Q \cdot R$, etc.). All we are being informed of by "$P \supset Q$" is that either Jones is not a bachelor or Jones is male. *In fact*, one or the other (or both) of these contingencies obtain. We are *not* here being modally constrained, as we would be were we informed that it *must* be the case that $\sim P \vee Q$—that any Jones *must* either be male or else must be other than a bachelor. Of the possibility of some Jones being neither male nor

other than a bachelor (e.g., a female bachelor) we are told only that this never in fact happens—not that it could not happen.

This sanctions the following inference:

$$[(P_{et} \dashv Q_{et})^n \dashv (\sim P_{et} \vee Q_{et})^c]^n$$

And this echoes example 2 above; "If the contingent truth of P entails the contingent truth of Q, then *in fact* either it is contingently true that Q or it is contingently true that ~P." Since it is necessary that every bachelor be male, then in fact every person consulted during the recent census was either male or not a bachelor (or both). Again, that every consulted person *must* be either male or not a bachelor goes beyond what "P ⊃ Q" actually claims; "P ⊣ Q" would be required to convey the idea that there could be no exception. So the claim "~P ∨ Q" must even be considered provisionally to have a consistent negation.[4]

Lewis and Langford write (*op. cit.*, page 137) that "whenever a strict implication can be asserted, the corresponding material implication can also be asserted. The converse does not hold; . . . the assertion of a strict implication is a stronger statement than the assertion of the corresponding material implication." The justification of this contention thus rests on assuming the validity of the cross-type inference just explored.

Any two randomly chosen propositions are such that either one materially implies the other or vice versa. Not so with entailment. Randomly chosen propositions *might* be logically independent. There must be a *deductive* connection between two assertions before they can conspire in an entailment. "P entails Q" means "It is impossible that P should obtain but not Q." "P materially implies Q," however, means only "It never *in fact* happens that P obtains when ~Q also obtains." Thus, it never happens that bachelors weigh more than two tons. Hence, being a bachelor materially

[4] This conclusion will seem inconsistent with Lewis and Langford, 18.7; $(P \dashv Q) = [\sim \Diamond \sim (P \supset Q)]$. No simple and satisfying resolution of this tension occurs to me. Since 18.7 entails $(P \dashv Q) \dashv (P \supset Q)$ and since this is equivalent to $(P \dashv Q) \dashv (\sim P \vee Q)$, the analysis we have tendered for the latter still seems to hold. Since $\sim P_{et}$ and Q_{et} are both contingent, their disjunction must also be contingent, at least in every case where $Q \neq P$.

In general, it would appear that in an inference such as $[(P_{et} \dashv Q_{et})^n \dashv (\sim P_{et} \vee Q_{et})^c]^n$—e.g., "Since every bachelor cannot but be male, then in fact every person consulted during the current census will either be male or not a bachelor"—we are moving from an intensional analysis of a term to its extensional analysis. The *vel* in the apodosis invites this interpretation. 18.7 in Lewis and Langford encourages one to overlook this shift, but, inasmuch as a very natural interpretation of $\sim P_{et} \vee Q_{et}$ is extensional, a very natural and wholly legitimate way of interpreting the inference before us would be as cross-type.

implies weighing less than two tons. But being a bachelor does not *entail* weighing less than two tons; "weighing less than two tons" is no part of the semantic content of "being a bachelor." One cannot deduce that Jones weighs less than two tons simply from his being a bachelor.

Thus, if Q necessarily follows from P (that is, $P \dashv\!\!\!\supset Q$), then, in fact, you will never discover P in the absence of Q (that is, $P \supset Q$).

That one can infer from a deductive claim to a factual claim is at the heart of the system of strict implication.

All of these putative cross-type inferences can be set out schematically as follows:

THE SCHEMATA OF CROSS-TYPE INFERENCES[5]

Contingent → Necessary	Necessary → Contingent
(1) $[(P_{ct} \cdot Q_{ct})^c \dashv\!\!\!\supset (P_{ct} \circ Q_{ct})^n]^n$	(1) $[(F_1F_2F_3F_4 \rightarrow Rc)^n \dashv\!\!\!\supset (F_4 \rightarrow Rc)^c]^n$ that is, "→ R" = ". . . is (a) good inductive reason for c . . ."
(2) $[(P_{ct})^c \dashv\!\!\!\supset (\Diamond P_c)^n]^n$	
(3) $[(\sim P_{ct})^c \dashv\!\!\!\supset (\Diamond \sim P_c)^n]^n$	
(4) $[(P_{ct} \cdot Q_{nt})^c \dashv\!\!\!\supset (Q_{nt})^n]^n$	(2) $[(P_c \dashv\!\!\!\supset Q_c)^n \dashv\!\!\!\supset (P_c \supset Q_c)^c]^n$
(5) $[(P_{ct})^c \dashv\!\!\!\supset (Q_{nt})^n]^n$ because $P_{ct} \dashv\!\!\!\supset (P_{ct} \cdot Q_{nt})$	

IV. ON RESOLVING APPARENTLY CROSS-TYPE INFERENCES

Our budget of cross-type inferences now spreads before us. Should they go unchallenged, they could conceivably bolster mystery-mongering metaphysics—and blunt the instruments of analysis. More important, they might even perplex us concerning the very nature of deduction.

Let us reconsider each inference in turn. It may be possible to draw the intellectual sting from them. Perhaps we can relieve the conceptual pain caused by these metaphysical progeny of the Ontological Argument, that most notorious of all cross-type inferences.[6]

1. "If it is contingently true that P and contingently true that Q, then the claim that P is contingently true and the claim that Q is contingently true are consistent."

[5] Again, remember that superscripts serve merely as comments upon the propositions preceding them. They are never part of the inference, but (as it were) logician's asides characterizing the previously stated inferences.

$(P_{ct} \vee \sim P_{ct})^n$ thus reads: "Either P is contingently true, or it is not the case that P is contingently true"; and the superscript "n" marks this disjunction as necessary, although this characterization is itself no part of what the disjunction under consideration asserts.

[6] "G necessarily has the properties α, β, γ, δ, and ϵ (for 'existence'); therefore *in fact* G has the property ϵ."

The guarantee of the validity of this inference is not logical, perhaps not even conceptual, in character. The guarantee is "ontological," although this is not to be misunderstood. We are being told that *the world* is such that a contingent proposition (P_{ct}) and its negation ($\sim P_{ct}$) cannot both be true at once. Or that Q (in Q_{ct} above) cannot take as its value $\sim P$. This is a reasonable claim. But I am not convinced that its guarantee is purely logical. The implication, that is, seems not to be strict. Perhaps the connection is at most one of material implication: *in fact*, there is no as-yet-discovered case where a contingent proposition (P_{ct}) and a contingent proposition (Q_{ct}) are simultaneously true, but where the claim that P_{ct} and the claim that Q_{ct} fail to be consistent. But, could one *deduce* their consistency just from the fact that P_{ct} and Q_{ct} are both contingently true? Why *not* say that P_{ct} and $\sim P_{ct}$ could simultaneously obtain? What is the nature of the *reason* for denying this, as most of us would? Why *not* suppose that the world could contain "contradictory" states of affairs? Why *not* suppose that an otherwise well-formed language could conceivably contain contingent descriptions of the form "P_{ct} and $\sim P_{ct}$"?[7] There *are* good reasons why we should not suppose this. But might it not be argued that these reasons go beyond any narrow theory of deduction? Such reasons are usually systematic, broadly conceptual, or even "metaphysical" in nature. We cannot, I submit, *establish* that nature does not at once embrace inconsistent facts simply by explicit definitions, linguistic conventions, and *modus ponendo ponens*. Presumably, it might be just barely possible, and barely intelligible, to dispute this issue. Hegel may be a case in point; his reaction to inconsistency was apparently distinguishable from that of most philosophers. This is not intended as a convincing reference. The onus must, of course, rest on the Hegelian to make *his* position intelligible. But, rather than simply lay down the issue against the possibility of his doing so, I should prefer to allow claims as to what is or what is not ontologically possible to rest ultimately upon considerations more "philosophical" than any narrow appeal to what deductive theory can or cannot sanction.

So, in fact, concurrently true contingent propositions may never yet have been discovered to be inconsistent. But, so far as I can see, this no more allows one to *deduce* their consistency from their

[7] "*The Mona Lisa is smiling* is contingently true, and *it is not the case that the Mona Lisa is smiling* is contingently true" might be a *prima facie* candidate for this status. Special semantical questions arise here which it is not our purpose to explore. But the sentential form of such an utterance is at least similar enough to the possibility mooted above to warrant some patience with what follows.

truth than would the historical fact that no bachelor has ever exceeded two tons allow us to deduce that Jones weighs less than two tons just from his being a bachelor. Jones's sex *can* be deduced from his being a bachelor. That he weighs less than two tons can be inferred from his being a bachelor. But one must distinguish deductive inferences like "that Jones is a bachelor entails that he is male" from inferences of a different kind, e.g., "that P and Q are each contingently true 'entails' that the contingent truth of P and the contingent truth of Q are consistent." This latter seems to me not to constitute genuine entailment. Unless special and arbitrary semantic conventions are adopted, can it really be represented as any more than a material implication? It seems to be no stronger an inference than that from lightning to thunder—and no weaker than that from sunspots to a wheat failure.

Now, my suggestion that P_{et} and Q_{et} may only *materially* imply the consistency of P_{et} and Q_{et} is open to the following counter-move:

If $[(P_{et} \cdot Q_{et})^c \supset (P_{et} \circ Q_{et})^n]^n$ is taken as a premise and $(P_{et} \cdot Q_{et})^c$ as another premise, then these two entail $P_{et} \circ Q_{et}$, which is necessary. But both premises are contingent; $P_{et} \cdot Q_{et}$ is certainly contingent. And $(P_{et} \cdot Q_{et})^c \supset (P_{et} \circ Q_{et})^n$ is no more than $\sim (P_{et} \cdot Q_{et})^c \vee (P_{et} \circ Q_{et})^n$—which, in turn, is equivalent to $\sim [(P_{et} \cdot Q_{et})^c \cdot \sim (P_{et} \circ Q_{et})^n]$. Since an internally consistent conjunction containing a contingency must itself be contingent, it follows that both premises are contingent. Yet the conclusion, $P_{et} \circ Q_{et}$, is necessary (either necessarily true or necessarily false; i.e., it is L-determinate). Hence, we have here yet another inference that looks cross-type, namely:

$$\{(P_{et} \cdot Q_{et})^c \cdot [(P_{et} \cdot Q_{et}) \supset (P_{et} \circ Q_{et})]^c\}^c \dashv (P_{et} \circ Q_{et})^n$$

Precisely what is involved in designating as "contingent" the material implication $(P_{et} \cdot Q_{et}) \supset (P_{et} \circ Q_{et})$, that is, $\sim [(P_{et} \cdot Q_{et}) \cdot \sim (P_{et} \circ Q_{et})]$, will be explored below in example 4. This is but a degenerate sense of contingency. Its effect on the general philosophical utility of the HLLL dictum is negligible, as we hope to disclose.

2. "That P is contingently true entails the claim that contingent P is logically possible."

In another place[8] the perplexities of this apparently cross-type inference are resolved somewhat as follows. The claim that P_{et} obtains is a contingent claim. The claim that P_{et} need not have obtained; it could have been false. However, that P is *contingent* is not a contingent claim. If true at all, it could not but have been

[8] Cf. Hanson, *Philosophical Studies*, October, 1959.

true. For this just *is* the claim that the negation of P (within some language L) is consistent. This we settle by reflection, not observation. Such is the teaching of Hume and Leibniz.

$\sim$P will be consistent whether P is contingently true or contingently false. So the assertion "P is contingently true" is composite. It consists of (1) a necessary component, viz., "P is contingent (true or false); that is, P and $\sim$P are consistent"; and further, of (2) a contingent component, viz., "contingent P is in fact true." That *P is logically possible* is inferred only from the necessary component (1). That P is logically possible just *is* the claim that P is self-consistent; i.e., entails nothing of the form $F \cdot \sim F$. (Cf. Lewis and Langford, 18.1.)

Hence, that P is logically possible is the claim that P is self-consistent. And that P is contingent (true or false) is the claim that P and $\sim$P are each self-consistent. The inference is thus only apparently cross-type. Its logical bones look like this: "P is self-consistent, therefore P is self-consistent." This is not problematic.

Even thus boiled down, however, our example does not constitute an equivalence. The conclusion, "P is logically possible," follows not only from P's being contingently true; it follows also from the claim that P is necessarily true. Hence, from "P is logically possible" one cannot infer that P is contingently true; P might be necessary. But from P's being contingently true (or false) it does follow that P is logically possible, i.e., self-consistent.

Remarking the composite character of the claim "P is contingently true" does not settle all present pretensions to cross-typicality. Let it be agreed that "P is contingent" is a necessary claim (true or false), and that "contingent P is true" is contingent (true or false). Still, there remains a sense in which the conclusion, viz., "P is logically possible," can be inferred from the conjunction of the first two, that is, "P_c is contingent *and* P_c is true." This entire conjunction is itself contingent; its truth cannot be determined by reflection. Nonetheless, that P is logically possible follows, by detachment, from the conjunction itself—or at least from one of the conjuncts. Thus (P is contingent *and* P is true)^c $\dashv$ (P is logically possible)^n. But this has now become an instance of a general case, to be examined below under example 4.

It might be argued that, although we have resolved these first two examples in different ways, they ought to have had the same analysis. Concerning example 1, we suggested that, at most, it constituted a material implication, not an entailment. Example 2 was felt to comprise a composite premise—from one component of which the conclusion followed necessarily. However, in 18.3, Lewis and Langford set out $(P \circ Q) = \Diamond (P \cdot Q)$; this was Lewis's defini-

tion of consistency in his *Survey of Symbolic Logic*. Since example 2 has the form $P \dashv \Diamond P$, substitution will give us $(P \cdot Q) \dashv \Diamond (P \cdot Q)$, which, by 18.3, gives us example 1, $(P \cdot Q) \dashv (P \circ Q)$.

Hence, 1 and 2 are really the same example. Either of our two resolutions should apply to each case. That is, it might be said that both, as they stand, constitute at most a material implication; *in fact*, the protases have never been found to hold where the apodoses have not also held. Or, if it is insisted that these are genuine entailments (as opposed to being mere material implications), the respective protases must be construed as being composite. Just as the claim "P is contingently true" is composite, so is the claim "P and Q are contingently true." These assert (a) contingency, and (b) truth; and (a) and (b) are not the same. In both examples the apodosis follows *only* from (a) the contingency (respectively) of P and Q (in example 1) and of P (in example 2). To say of a proposition that it is contingent is to make a *necessary* claim about it (i.e., either necessarily true or necessarily false). Therefore, if we insist that these two examples constitute genuine entailments, then the apodoses can follow only from this necessary component (a) in the protases. $P \circ Q$ is the same as $\Diamond (P \cdot Q)$. And, in our examples, this follows directly from the claim that $P \cdot Q$ is contingent, that is, from $\Diamond (P \cdot Q) \cdot \Diamond \sim (P \cdot Q)$. So, in examples 1 and 2, if the protases are *not* composite, the inference can be no stronger than material implication. But, if the inference *is* stronger, i.e., is an entailment, then the protases must be composite.

3. "That $\sim P$ is contingently true entails the claim that it is logically possible that $\sim P$."

All the considerations just explored in 2 above apply in this case.

4. "If it is true both that P_{ct} *and* that Q_{nt}, we can from this conjunction infer that Q_{nt}."

To repeat: The conjunctive premise is contingent. Its truth cannot be assessed by reflection alone. The inference is valid; it embodies only the rule of detachment. How is this to be squared with the HLLL dictum? The consequences of leaving it unchallenged could be serious. For, if it stands, no *general* philosophical objection to inferring necessary conclusions from contingent premises can be tenable; no *general* attack on a kind of argumentation exemplary in speculative metaphysics can succeed.

Notice this: The characterization ". . . is a necessarily true proposition" covers (at least) two different kinds of reference. This is well known. That all bachelors are male is necessarily true within our language L, in that its semantic content renders its

negation inconsistent, *à la* Hume and Leibniz. However, that no surface is simultaneously pink and purple all over will also be necessarily true within the same L—but not for the same reason as the bachelor example. It is not so much that the negation of this color-proposition is easily reducible to inconsistency; this may not even be possible. Still, this proposition's negation is conceptually untenable—unintelligible. That there is a surface at once all purple and all pink is only an *apparent* proposition. It gives us nothing whatever to entertain. No coherent conception answers such a description. Another example of this "weaker" necessity might be "There are no *perpetua mobilia*." The grammatical negation of this, within the appropriate L, will express an apparently consistent claim. But it is pretty certain that no one now has any clear, scientifically articulable idea of what a *perpetuum mobile* (First Type) would be like. Hence, since "There are no *perpetua mobilia*" is a claim that now lacks an intelligible negation (in that no one can give semantic sense to the physics of this negation), this justifies calling it a necessary claim. It is invulnerable, even though formally deniable.

Could the class of contingent statements also be subdivided? Can it be said that (1) "No bachelor exceeds two tons" is contingent in *just* the way (2) "No bachelor exceeds two tons *and* no bachelor is female" is contingent? Both are contingent in that establishing their truth requires more than reflection. Indeed, one way of negating (2) might even be said to be consistent, but surely not in the way in which that of (1) is.

The denial of "$P_{ct} \cdot Q_{nt}$," moreover, has some extraordinarily peculiar properties, properties never encountered in "ordinary" contingent statements. Presumably, one could deny that $P_{ct} \cdot Q_{nt}$ either because it is false that P_{ct} or because it is false that Q_{nt}—or both. If "$P_{ct} \cdot Q_{nt}$" is denied because it is false that P_{ct}, the situation is not novel. Were the facts different, "$P_{ct} \cdot Q_{nt}$" would then express a contingent truth. But when "$P_{ct} \cdot Q_{nt}$" is rejected not because "P_{ct}" expresses what is factually false, but because "Q_{nt}" expresses what is logically false, then the situation becomes bizarre. For in this case one could apparently convert a contingently false conjunction (viz., $P_{ct} \cdot Q_{nt}$) into a contingently true one not by altering facts or by denying contingent claims, but by negating an inconsistency. In a long conjunction of genuinely contingent propositions, some of which are false, one could not know *which* to negate without making observations. When it is only an inconsistent conjunct to these that needs alteration, however, no observation is required. Symbol inspection will readily determine the miscreant proposition. But when two genuine

contingent propositions are conjoined to form an "ordinary" factually false conjunction, symbol inspection is never sufficient for determining which of the two is false.

Hence, just as there are two distinguishable senses of ". . . is necessary," so also (although in a different way) there may be two distinguishable senses of ". . . is contingent." "$P_{ct} \cdot Q_{ct}$" *is not contingent in the same sense as is* "$P_{ct} \cdot Q_{nt}$."

Moreover, there is some *conceptual impropriety* in referring to a conjunction of a contingently true proposition and a contradiction as contingently false. To say that "Today is Friday *and* some bachelors are married" is *contingently* false, sounds queer, to say the least. The impropriety consists in its being clear that the falsity within such a conjunction lies not in the contingent component *per se*, but rather in the fact that a contradiction has been conjoined to that component, forming thereby an unacceptable conjunction. So, let us provisionally distinguish a hard sense of contingency from this merely "logical" sense. Distinguish the sense in which it is contingently false that the earth has two moons from the sense in which it is "contingently false" that the earth has one moon *and* some bachelors are married.

Finally, that a conjunction of thirty propositions might be adjudged *contingently* false because one of the conjuncts was *inconsistent* would not, as a matter of linguistic practice, ever be entertained seriously.

It might be countered that this analysis is unnecessary since there is *no* sense in which a conjunction of a contingently true claim and a contradiction is contingently false. Perhaps such a conjunction generates *only* an impossible (i.e., self-inconsistent) proposition. There are serious objections to this line. There is no way of characterizing $P_{ct} \cdot Q_{nt}$ as true other than by calling it "contingently true." It cannot be necessarily true. Thus, analogously, "All bachelors are male, *and* there are in fact bachelors" can at most be contingently true. Its truth cannot be established by reflection alone. But then, if $P_{ct} \cdot Q_{nt}$ is contingently true, there must be some sense in marking $\sim (P_{ct} \cdot Q_{nt})$ as contingently false. If P is contingent, $\sim P$ is also contingent; and $P_{ct} \cdot Q_{nt}$ is a legitimate substitution for P.

If it counts as a logical principle that

(a) a contingent conjunction will be false when either conjunct is false, and that

(b) if R is contingently true, then $\sim R$ is contingently false, and that

(c) if $R = (P_{ct} \cdot Q_{nt})$ and $\sim R = \sim (P_{ct} \cdot Q_{nt})$, where it remains true that P_{ct},

then, granting all this, it is certainly reasonable to suggest that, when Q_{nt} is false, then $P_{ct} \cdot Q_{nt}$ is contingently false. Just to lay it down that, here, $P_{ct} \cdot Q_{nt}$ *must* be characterized as impossible, would be to knife through a Gordian knot of concepts. A natural extension of established logical principles has resulted in a case for calling $P_{ct} \cdot Q_{nt}$ "contingently false" whenever Q_{nt} is false. Furthermore, we have given *reasons* for resisting this natural extension. But simply to *rule* that, here, $P_{ct} \cdot Q_{nt}$ is impossible is not giving reasons. Hence, it constitutes no proper analysis of the case before us.

This distinction between types of contingencies, if it is a sound one, is not merely a scholastic proliferation of minute differences, lacking in any philosophical motive. Rather, it provides just what is needed to mark the notion of contingency involved in our first set of examples as degenerate and conceptually harmless.

Nonetheless, since the entailment before us is formally sound, one cannot avoid the responsibility of modifying the HLLL dictum. It would appear to be an analytic philosopher's obligation to block all cross-type inferences save those which are logically degenerate and, hence, conceptually harmless. It may be theoretically quite difficult to express this exception in a general way. But the attempt will have to be undertaken.

5. "P is contingently true; *so* P is contingently true and Q is necessarily true." That is, $P_{ct} \dashv (P_{ct} \cdot Q_{nt})$.

This ultimately sanctions "P is contingently true; *so* Q is necessarily true," that is, $P_{ct} \dashv Q_{nt}$. But this is only one of the "paradoxes" of strict implication. Since a necessary proposition (Q_{nt}) is implied by any proposition whatever, it is certainly implied by *any* contingent proposition (including P_{ct}). Such an inference *would* be cross-type. But it would also be obviously degenerate, even by one of the leading conceptual principles of strict implication. P_{ct} and Q_{nt} may be thoroughly independent propositions, between which no one would ever suppose any deductive connection to obtain. The *raison d'être* of Lewis and Langford's system is to give sense to the idea of independent propositions, between which no entailment will hold—even though some material implication may hold. Yet, even within this system, Q_{nt} will be entailed by any P whatever; *ergo*, $P_{ct} \dashv Q_{nt}$. Perhaps this reflects more on the LL system of strict implication than on the general status of cross-type inferences.

Concerning Inferences from Necessity to Contingency

Logicians are more likely to feel that this variety of inference is clearly invalid—a feeling not generally held with respect to our

earlier examples. In addition to the argument immediately preceding section II of this paper, the following one might also be be convincing:

1. P is necessary	$\sim \Diamond \sim P$
2. Q is contingent	$\Diamond Q \cdot \Diamond \sim Q$
3. P entails Q	$P \dashv Q$ that is, $\sim \Diamond (P \cdot \sim Q)$
4. Necessities entail only necessities	$[(P \dashv Q) \cdot (\sim \Diamond \sim P)] \dashv \sim \Diamond \sim Q$[9]
5. ~Q is possible	$\Diamond \sim Q$ (from 2)
6. Q is necessary	$\sim \Diamond \sim Q$ (from 1, 3, 4)
7. ~Q is both possible *and* impossible	$(\Diamond \sim Q) \cdot (\sim \Diamond \sim Q)$ (from 5, 6)

Reductio ad absurdum.

In the light of this demonstration it becomes difficult to construct plausible candidates for the title "Inference from Necessity to Contingency." Nonetheless, our two examples are probably about as strong as could be found.

1. "(a) 'That F_1, F_2, F_3, F_4 obtain is good inductive reason for expecting C.'

   entails

   (b) 'That F_4 obtains is *a* good inductive reason for expecting C.' "

Since this putative cross-type inference has been dealt with elsewhere,[10] the problem will not be engaged in detail.

Although an attractive candidate for cross-typicality, this is parasitic on ambiguities in "is a good inductive reason." On one interpretation of this expression, what (a) expresses *does* entail what (b) expresses. But then (b) is also necessary; i.e., the inference does not cross types at all. On another interpretation, what (b) expresses *is* contingent, but then there is no entailment at all between (a) and (b).

One way of expanding (b) so that its semantic content spreads before us, is this:

F_4, when construed as a premise together with F_1, F_2, and F_3 (or some other suitable set), is such that conceptual paralysis would result from asserting all these premises (F_1 to F_4) and at the same time denying that we have good inductive reason for expecting C. This means that any one of these premises, for instance, F_4, necessarily *must* be *a* good reason for C; it constitutes part of the *meaning* of ". . . having good inductive reason for expecting C."

Now the inference is horizontal: from a necessary claim to another necessary claim. No more is said here than would have been said had we argued:

[9] Lewis and Langford, 18.53. This step may beg the very question at issue. But since it would seem to be part of the apparatus of *any* modal logic, I can see no reason not to use it here.

[10] Cf. Hanson, "On the Impossibility of Any Future Metaphysics," *Philosophical Studies*, December, 1960; "Good Inductive Reasons," *Philosophical Quarterly*, 1961.

(a′) "That P and P ⊰ Q obtain is good deductive reason for Q"

entails

(b′) "That P ⊰ Q obtains is *a* good deductive reason for Q."

Here too, *if* (b′) is entailed by (a′), (b′) must be necessary. Otherwise, (b′) is just contingently false; P ⊰ Q, by itself, is not *in fact* a good deductive reason for Q.

Another, alternative, way of expanding (b) is this:

There are in fact certain contingently true claims (F_1, F_2, F_3), such that, in conjunction with F_4, they actually do give us good inductive reason for expecting Q.

This could hardly be more than contingent. There *need not have been* true statements F_1, F_2, F_3, giving us good inductive reason for C (there need not have been any statements at all)—but *in fact* there are.

But now there is no entailment whatever between (a) and (b). (a) remains necessary; it sets out the semantic conditions for having good inductive reason for expecting an event. However, now (b) states the fact [wholly independent of (a)] that in our world *there are* conditions F_1 to F_3 which, when conjoined with F_4, *do in practice* warrant our inference to C. This does not follow from (a) at all; (b) might remain true even were (a) demonstrated (e.g., by a logician) to be self-contradictory.

In general, then, what begins apparently as a startling cross-type inference ends as an understandable confusion at best, and a disguised ambiguity at worst. Deductive theory is done no harm by this example, and speculative metaphysics is certainly done no good.

2. Concerning now the inference,

$$[(P_{et} \dashv Q_{et})^n \dashv (P_{et} \supset Q_{et})^n]^n$$

this is not so devastating as at first it appears.

One *could* just say, e.g., that Lewis and Langford's system must be ill founded, since it sanctions this cross-type inference. But this might be a *petitio principii*; Lewis and Langford have been useful in all our appeals thus far. Why jettison the system when the very rules we have been relying upon turn up something uncomfortable? There is a less *ad hoc* line of attack.

We have observed that $\sim P_{et} \vee Q_{et}$ will itself be contingent; its negation, $\sim (\sim P_{et} \vee Q_{et})$, would at least be consistent. Thus, "That it is contingently true that 'The moon is made of limburger cheese' materially implies that it is contingently true that 'The earth is flat'," *means* only that *either it is not the case that "The moon is made of limburger cheese" is contingently true, or else "The earth*

is flat" is contingently true—and this disjunction is certainly contingent. The negation of this is also in principle contingent. But who would ever recast this as a *strict* implication, e.g., "That the earth is flat is *deducible* from the moon's being made of limburger cheese"? There is no such strict implication. "P $\dashv\!\!\!\supset$ Q" means that $\sim \Diamond (P \cdot \sim Q)$. But it *is* possible, however, that the moon should be made of limburger cheese while the earth is other-than-flat. The entailment relation cannot hold here, therefore; although a material implication can.

So this peculiar inference would never even come up for attention save when P $\dashv\!\!\!\supset$ Q does actually hold—i.e., when Q *is* deducible from P. But then our problem is one we have already faced: If P $\curlyvee$ Q is necessary, then, of course, P $\dashv\!\!\!\supset$ Q *does actually obtain*, whatever that may mean (i.e., P $\supset$ Q is factually true).

This is either an exploitation (for "philosophical" purposes) of some degenerate sense of contingency *or* an elaborate way of remarking one of the paradoxes of material implication.

That is, since P $\supset$ Q *must* be true when its values are chosen so that P $\dashv\!\!\!\supset$ Q might just as well have been asserted, why then naturally P $\supset$ Q follows from P $\dashv\!\!\!\supset$ Q. In this case, however, it is difficult to construe "P $\supset$ Q" as being contingent at all. A related point is that, whenever P $\supset$ Q is true, it follows (as one of the paradoxes of material implication) from any proposition whatever. *Eo ipso*, it follows from P $\dashv\!\!\!\supset$ Q. Nothing of special philosophical interest emerges.

Suppose now that someone reasoned: "That the moon is made of limburger cheese (M), is deducible from the earth's being round (E)." He argues, in effect, that E $\dashv\!\!\!\supset$ M. This is just the claim that $\sim \Diamond (E \cdot \sim M)$. A critic might try to reveal the error by deducing E $\supset$ M from E $\dashv\!\!\!\supset$ M. Since this is the same as deducing $\sim$E $\vee$ M, that is, "Either the earth is not round, or the moon is made of limburger cheese," and since this latter disjunction is actually *false*, the critic might succeed thereby in demonstrating the untenability of the original argument, viz., E $\dashv\!\!\!\supset$ M.

Would one have been endorsing a genuine, nondegenerate cross-type inference [namely, (E $\dashv\!\!\!\supset$ M) $\dashv\!\!\!\supset$ (E $\supset$ M)] in trying thus to expose the original error?

No. No more than would a logic teacher be endorsing a fallacy by writing "If Jones is Welsh then he is human; he is human, therefore he is Welsh" *in order to demonstrate*, by this unacceptable conclusion, the unsoundness of the argument. We often *follow through* on an inference to reveal its shortcomings. This does not signal acceptance of that inference. Similarly, although we may follow through with $(P_c \dashv\!\!\!\supset Q_c) \dashv\!\!\!\supset (P_c \supset Q_c)$ to shake confidence in

$P_o \supset Q_o$, this does not rob us of the capacity to criticize the *general* status of the inference.

Both these inferences remind one of the ancient inference-schemata: "P is necessary, therefore *in fact* P," and "P is impossible, therefore *in fact* ~P." Logicians of many ages and of many persuasions have been seduced into thinking that, in virtue of such inference-patterns as these, logical necessities might be thought to be factually efficacious. Letter-day Aristotelians, intentional logicians, and Idealist thinkers have felt it somehow to be informative about our world to be told that, for instance, $P \vee \sim P$ obtains in it.

Thus: "It is impossible to square the circle; hence *in fact* no one in America, during 1960, found a quadrature of the circle." But the apodosis of this claim cannot be *merely* contingent, inasmuch as the very ideal of an American in 1960 actually succeeding in squaring the circle is not even an intelligible idea. In the remote past the notion of a constructive quadrature of the circle could never have been more than a radically confused idea. Just putting the expression "in fact" before the apodosis will not infuse the latter with factual content. The world becomes no richer epistemologically just because some logician moves through it announcing "*In fact* all triangles have three vertices," "*In fact* all bachelors are male," "*As a matter of fact* in our world, either a proposition or its negation is true," etc. A claim that excludes no possibility whatever cannot convey anything factual. It cannot express anything logically novel. At most, its message may be psychologically new, as when we learn, e.g., that transfinite addition is possible.

A claim like $P \vee \sim P$ can appear to be informative only when one conflates (1) the claim that it is contingent whether or not it is P, or ~P, that does in fact now obtain, with (2) the logical fact that $P \vee \sim P$ exhausts all possibilities for anything whatever obtaining. Only by confounding the observations necessary to determine whether it is P, or ~P, that *does* obtain, with the distinct fact that no third possibility beyond these two can exist, only so can $P \vee \sim P$ appear to have any factual content.

One is reminded of the story told of G. E. Moore, who, happy with his wife's having given birth, was stopped by a well-wisher and asked: "Is the child a boy or a girl?" To this, Moore answered, "Yes!" His friend was asking for information about a contingent matter. But, without further punctuation, his question begged for Moore's "tautological" interpretation. Only by conflating Moore's answer with the obvious intent of the question could one construe this as the expression of a necessary fact. Only by confounding the reflection that babies are either male or female with the

further fact that it requires observation to determine which, could one suppose that here is a description of fact which is invulnerable. In this case we have a transparent howler before us. Treating "P $\vee$ $\sim$P" as if it were both necessary *and* informative may not be a transparent howler, but it is at least translucent.

Certain "realistic" discussions of the nature of logic make their case largely by forcing these distinct considerations onto the same track. Such discussions have not yet realized, apparently, that *invention is the mother of necessity.*

V. Restating the HLLL Dictum

In the history of philosophy some cross-type inferences have been harmful. The subhistory of speculative metaphysics is replete with examples. It was the HLLL intention to design a criterion against which these miracles of argument (which, since they can accomplish virtually anything, actually accomplish nothing) might be exposed for what they are. However, there are inferences, in theoretical logic and in everyday reasoning, which apparently *do* proceed across types. It has been our purpose to provide some understanding of these. Our verdict is that they are all conceptually harmless. They are harmless either because they rest *not* on logical principles alone, but on wider metaphysical, systematic assumptions, *or* because they trade on degenerate inferences, of which P $\supset$ P might be taken as an example; the "paradoxes" of material and strict implication provide others. From such as these, speculative metaphysicians gain nothing, and deductive logicians lose nothing.

Restating the HLLL dictum can be achieved, therefore, as follows:

From what is contingent nothing necessary follows, save only in harmless and degenerate cases. These exceptions exist only by there being a weak, merely logical sense of "contingent" quite distinct from what the designers of our "dictum" ever had in mind.

And from what is necessary nothing contingent follows, save again in those harmlessly degenerate cases in which "contingent" takes on a semantic force totally unlike what is at issue in the HLLL dictum.

Any *unquestionable* example of a contingent statement remains such that it neither entails nor is entailed by any *unquestionable* example of a necessary statement.

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THE LOGIC OF THE ONTOLOGICAL ARGUMENT

PROFESSOR Malcolm's recent exposition,[1] correct so far as it goes, of Anselm's second ontological argument (there are indeed two) can be partly formalized in a chain of valid inferences as follows:

1. $(\exists x)Px \rightarrow N(\exists x)Px$ The existence of perfection can only be necessary.

2. $\sim N(\exists x)Px \rightarrow N \sim N(\exists x)Px$ The non-necessity of perfection entails its necessary non-necessity (modal status, in the absolute logical sense, is always necessary).

3. $N \sim N(\exists x)Px \rightarrow N \sim (Ex)Px$ The necessary falsity of the consequent entails the necessary falsity of the antecedent (1 and a modal form of modus tollens).

4. $N(\exists x)Px \vee N \sim (\exists x)Px$ Either perfection exists necessarily or its existence is impossible (taut. and 2, 3).

5. $\sim N \sim (\exists x)Px$ The existence of perfection is not impossible.

6. $N(\exists x)Px$ Perfection exists necessarily (4, 5).

Concerning 1. For an individual, to exist contingently is an imperfection; hence no individual so existing can exemplify perfection. Malcolm has well explicated this principle, Anselm's great discovery; so has J. N. Findlay, from an anti-theistic point of view.[2] Well or ill, I have expounded it on various occasions, without rebuttal from my philosophical colleagues.[3]

There is some difficulty about the interpretation of the variable x. The necessity, formally regarded, requires only that there be some individual or other exemplifying perfection. However, it is an old insight that perfection characterizes a unique individual, not a class of possible perfect beings. There can here be no *principium individuationis*. Hence, if perfection were capable of non-existence, we should have to admit what logicians rightly regard as absurd, a merely possible yet individual entity. Thus the necessary connection between perfection and uniqueness is an additional reason supporting 1. There are still others, which I here pass over.

2 applies a fairly standard axiom in modal logic. I do not

[1] Norman Malcolm, "Anselm's Ontological Arguments," *Philosophical Review*, 69: 41–62 (January, 1960).

[2] See Findlay's brilliant essary, "Can God's existence be disproved?," *Mind* (1948). Reprinted in A. Flew and A. Macintyre, *New Essays in Philosophical Theology* (London, 1955), pp. 47–56.

[3] For example, in Hartshorne and W. L. Reese, *Philosophers Speak of God* (Chicago, 1953), pp. 96–106, 134–37.

believe that the objections some have made to this axiom are here relevant.

4 infers the disjunction of the consequents of the tautological disjunction $(\exists x)Px \vee \sim (\exists x)Px$ and its implications via 2, 3.

5 is the most problematic assumption. Its justification lies outside the ontological argument proper. However, even without 5, the reasoning proves that perfection cannot contingently fail to exist and cannot contingently exist; it must either exist necessarily or be logically incapable of existence. "Empirical theism" and "empirical atheism" are alike illogical.

Malcolm's attempts to justify 5 are weak. Worse than that, he fails to rebut Findlay's objection that the argument appears to derive the concrete from the abstract, from a mere conceptual definition. Since in the divine omniscience the entire riches of all reality must be comprised, if the necessary existence of perfection is identical with the divine actuality, we face the contradiction that the more follows from the less, the concrete from the abstract. Malcolm's rebuttal, through an attempted distinction between "concrete" and "contingent," does not meet the difficulty. God's reality must be more than can be contained in a mere empty definition, couched in general terms like "that," "than which," "nothing," "greater," "can be conceived" (or however the term "perfect" is defined). Besides, it is evident upon reflection that the necessary can logically be but the common denominator of all possibilities, and what is that if not something highly abstract? A necessary proposition, as Lewis said, is entailed by any and every proposition, even the most abstract and empty: thus its meaning can only be the most universal or abstract element in the meanings of propositions in general. The concrete reality of God cannot be such an abstraction, at least if God is the proper object of worship.

What is the conclusion? Findlay says (I interpret him rather freely) that God cannot exist, since his existence must be necessary and, therefore, would represent the absurdity of a most abstract thing that at the same time was concrete.

There is another possible conclusion: that God is both necessary and contingent, hence both abstract and concrete, or that He has an essential but abstract character which is bound in any possible state of affairs to be actualized or concretized *somehow*, the how being always contingent. This is not the most usual conception of deity; indeed it is hard to find it prior to Socinus, and it has been somewhat rare down to our own time. I have argued in various places that this conception alone can remove the paradox of the step from the abstract to the concrete which is the real difficulty with the ontological argument. For according to the "Socinian"

view, the bare "existence of God" as necessary is utterly abstract, since it says only that the essence "perfection" is somehow concretely actual, no matter how. This amounts to saying, to put it in one of numerous equivalent ways, that whatever world there may be, it is fully and infallibly known and sustained by the divine power. Let the world be anything you please concretely, and let the particular state of infallible knowing of that world take any shape it can, so long as the mere form of infallibility is embodied, God exists. Infallibility is compatible with any content whatever; it is an empty form, in itself. Just such an empty—though, as we have seen, individual—form is perfection, in abstraction from its contingent modes of actualization with respect to contingent contents.

The justification of 5 cannot be adequately affected without revising the idea of God as above suggested, and one must also employ forms of theistic argument other than the ontological; to this extent the ontological argument is not self-contained. On the other hand, quite without 5, the argument can come to the aid of the other proofs, for what these need from the ontological proof, as Kant's analysis showed, is merely the exclusion of contingency from the existence of perfection, this existence meaning its being concretized somehow. In turn, to effect this exclusion, neither 5 nor the other proofs are required. There need not, then, be a vicious circle in employing all the proofs together. However, the other proofs, too, and the definition of "God" to be supported by them, must be correspondingly altered, somewhat as we have altered the ontological proof and its conclusion.

It will be a splendid achievement if Malcolm's fine article causes the philosophical world to take a fresh look at what has, for many decades now—but deceptively—appeared as a well-solved problem. Quite the contrary, there are many unexplored possibilities for its further elucidation; only a book could do any justice to them. An article can but hint at the subtleties of this topic.

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BOOK REVIEWS

Opere complete. GIOVANNI GENTILE. (A cura della Fondazione Giovanni Gentile per gli studi filosofici). Volumes XXXI–XXXIV: *Le origini della filosofia contemporanea in Italia* (Vito A. Bellezza, ed.). Firenze: Sansoni, 1958. xi, 422; 413; 378; 256 pp. Each volume, L. 2500. Vol. XXXVII: *Discorsi di religione* (4th ed.). Firenze: Sansoni, 1958. xii, 172 pp. L. 1800.

Giovanni Gentile was a thinker who would be no comfort to those who like to believe that a man can be a good historian of philosophy without being a good philosopher, or vice versa. The above volumes, though forming only a small part of Gentile's *Complete Works* (the Sansoni Collection is to consist of some sixty volumes), are quite typical of his way of thinking and can serve to indicate why a philosopher's historical and systematic contributions are difficult to separate in reality, however easy they may be to distinguish in theory. In fact, just to complicate matters, it should be added that, in the particular case of Gentile himself, it is difficult, not to say impossible, to differentiate the two kinds of contributions even in theory, given his neo-Hegelian thesis of the identity of history and philosophy.

Be this as it may, students of Italian philosophy should be grateful to the Gentile Foundation and the publisher for making available in handsome cloth-bound form the most extensive of Gentile's historical works, *Le origini della filosofia contemporanea in Italia.* We should also be grateful to Sig. Bellezza (the editor who, by the way, is Gentile's "bulldog") for using as text the original articles published from 1903 to 1914 in Croce's great journal, *La Critica,* rather than the more condensed material published in book form from 1917 to 1923.

As the present reviewer sees it, the significance of this four-volume work, which covers the period of Italian philosophy in the second half of the last century, lies not so much in that it takes us back to the origins of contemporary philosophy in Italy; its significance lies more in that the author there takes us back to the origins of his own philosophy, "actual idealism." Whether Gentile is discussing one group or other of the late nineteenth-century philosophers—Platonists, positivists, neo-Kantians, Hegelians—it is pretty clear from his comments and criticisms whither he himself is moving.

Turning to the other work under review, the importance of

the fourth edition of the *Discorsi di religione* lies in another direction. Here we get directly at the very heart of the Gentilian philosophy, with its emphasis on the "mystical" nature of the religious and on the "religious character of Italian idealism." But the most telling piece of the whole book is the one entitled "La mia religione" (pp. 123–144), which originated as a lecture at the University of Florence in February of 1943.

This lecture turns out to be, in retrospect, Gentile's last testament. In it he openly declares himself to be not only a Christian but a Catholic as well. However, he is quick to qualify his statement by adding that, just as he is his own kind of idealist in philosophy, so he is his own kind of Catholic in religion. To justify his stand, he appeals to a principle of his favorite Italian philosopher of the early nineteenth century, Vincenzo Gioberti, which is "poligonia del vero."

What irony that the man who defended the "many-sidedness of truth" and espoused an open-door policy to its attainment should have forgotten, by becoming spokesman for Fascism, his own insight! Or would Gentile contend, again on the same ground, that he was his own kind of Fascist in politics?

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Giordano Bruno. AUGUSTO GUZZO. Torino: Edizioni di *Filosofia,* 1960. 279 p. (*Scritti di Storia Filosofia.*) L. 2000.

This is an enlargement and completion of the author's *I Dialoghi del Bruno,* which first appeared in 1932 and which constitutes the central portion (Chapters V–XI) of the present volume. Professor Guzzo now gives us an adequate study of Giordano Bruno both as a character and as a philosopher.

Preceding the account of Bruno's *Dialogues* is a biographical sketch including substantial selections from the autobiography which Bruno composed for the Inquisition at Venice and which led to his eight years of imprisonment and to his martyrdom in Rome in 1600 when he was fifty-two years of age. The story of his career gives a lively portrait of a restless, bold, adventuring mind typical of the Renaissance, and of a freedom-loving monk, who abandoned the life and devotions of a *religiosus* to live as a wandering exile and as composer of some of the most impressive literature of the Copernican revolution.

The early works, written in Latin, reflect Bruno's excitement more than they do his philosophy. His exploration of the "art

of memory," his comedy, *Candelaio,* and his opening of the great seals, *Sigillus Sigillorum,* are youthful prologues to what he hoped would be his *Clavis Magna.* As the new world opened to his eyes and to his intellectual vision, he began to preach passionately his Epicurean theme, "Per tanto cambiar natura è bella." Among the entertaining and instructive incidents of Bruno's wanderings, an incident which Guzzo treats well and critically, was his brief attempt to find refuge among the Calvinists of Geneva. It took him only a few weeks to discover that he must flee Geneva for much the same reasons that had led him to flee from the Dominican convent near Naples. Bruno was clearly not an ecclesiastical Protestant, but rather a protesting philosopher of nature who, when he left the convent, also, as he put it, "abandoned religion." Guzzo points out that in Bruno's ethics there is a minimum of gladness and a minimum of sadness; and he cites a marginal comment by George Santayana in his copy of Guzzo's earlier work: "Bruno never got beyond distraction. Liberation for him was only a change of prisons" (p. 10).

The major works, the *Dialogues,* are taken up in chronological order. Guzzo tries to show how the problems raised in each lead into the next, so that the series leads to a philosophical culmination even though it lacks unity and system.

The concluding chapters of Guzzo's work (XII–XVI) give an account of: the philosophical aspects of Bruno's works on mathematics and magic (especially his attempt to indicate the various ways in which beings "attract" and "bind" each other into a cosmic One); his Latin poems, *De Minimo, De Monade,* and *De Immenso;* his trials by the Inquisitors; and his theory of God as "the nature of nature and the life of life."

The five-page Note at the end of the volume surveys the evaluations of Bruno during the seventeenth and eighteenth centuries.

Guzzo's work seems definitive and certainly supplants the earlier work of Spaventa. It is indispensable for any student of Bruno, and fascinating for any student of natural philosophy. I hope we may soon have an English edition, though it would be still better if all could enjoy the fine Italian.

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